

Various Facets of Spacetime Foam

Y. JACK NG^{1,2*}

¹ *Department of Physics & Astronomy, University of North Carolina,
Chapel Hill, NC 27599-3255, USA*

² *European Southern Observatory, Karl-Schwarzschild-Str. 2,
85748 Garching, Germany*

Abstract: Spacetime foam manifests itself in a variety of ways. It has some attributes of a turbulent fluid. It is the source of the holographic principle. Cosmologically it may play a role in explaining why the energy density has the critical value, why dark energy/matter exists, and why the effective dynamical cosmological constant has the value as observed. Astrophysically the physics of spacetime foam helps to elucidate why the critical acceleration in modified Newtonian dynamics has the observed value; and it provides a possible connection between global physics and local galactic dynamics involving the phenomenon of flat rotation curves of galaxies and the observed Tully-Fisher relation. Spacetime foam physics also sheds light on nonlocal gravitational dynamics.

Introduction

Unity of physics dictates that various physical phenomena and the principles underlying them are related to one another. But some of the concepts, phenomena and structures found in physics are more fundamental than others. I believe spacetime foam (arising from quantum fluctuations of spacetime) belongs to the first (fundamental) category. In this talk I will show that spacetime foam has a multiplicity of sides and will argue ¹ that it is the origin of some of the various phenomena we see around us. Spacetime foam manifests itself in the holographic principle. Its physics calls for a critical cosmic energy density and the existence of dark energy/matter.

* yjng@physics.unc.edu

¹Some of the interpretation of the physics given here may deviate from the original works I did with my various collaborators. I alone am responsible for such a reinterpretation.

At least partly it explains the observed critical galactic acceleration and it provides an intriguing dark matter profile. It has some attributes of a turbulent fluid. And its physics may be related to the nonlocality of gravitational dynamics. Each of these various facets of spacetime foam will be discussed in a separate section below.

But first, let us examine how foamy spacetime is, or, in other words, how large quantum fluctuations of spacetime are. This can be done by using the following two methods.

- The Wigner-Salecker experiment [1, 2, 3, 4]

To quantify the problem, let us consider the fluctuations of a distance l between a clock and a mirror. By sending a light signal from the clock to the mirror and back to the clock in a timing experiment, we can determine l . The clock's and the mirror's positions jiggle according to Heisenberg's uncertainty principle, resulting in an uncertainty δl . From the jiggling of the clock's position alone, the uncertainty principle yields $(\delta l)^2 \geq \hbar l / mc$, where m is the mass of the clock. On the other hand, the clock must be large enough not to collapse into a black hole; this requires $\delta l \gtrsim 4Gm/c^2$, which combines with the requirement from quantum mechanics to yield $(\delta l)^3 \gtrsim 4ll_p^2$ (independent of the mass m of the clock), where $l_p = \sqrt{\hbar G/c^3} \approx 10^{-33}\text{cm}$ is the Planck length. We conclude that *the fluctuation of a distance scales as its cube root* [5]:

$$\delta l \gtrsim l^{1/3} l_p^{2/3},$$

where we have dropped multiplicative factors of order unity. Henceforth we will continue this practice of dropping such factors except in a couple of places.

- Mapping the geometry of spacetime[6, 7]

Let us consider mapping out the geometry of spacetime for a spherical volume of radius l over the amount of time $T = 2l/c$ it takes light to cross the volume. One way to do this is to fill the space with clocks, exchanging signals with the other clocks and measuring the signals' times of arrival. This process of mapping the geometry of spacetime is a kind of computation, in which distances are gauged by transmitting and processing information. The total number of operations, including the ticks of the clocks and the measurements of signals, is bounded by the Margolus-Levitin theorem [8] in quantum computation, which stipulates that the rate of operations for any computer cannot exceed the amount of energy E that is available for computation divided by $\pi\hbar/2$. This theorem, combined with the bound on the total mass of the clocks to prevent black hole formation, implies that the total number of operations that can occur in this spacetime volume is no greater than $2(l/l_p)^2/\pi$. To maximize spatial resolution (i.e., to minimize δl), each clock must tick only once during the entire time pe-

riod. If we regard the operations partitioning the spacetime volume into “cells”, then on the average each cell occupies a spatial volume no less than $(4\pi l^3/3)/(2l^2/\pi l_p^2) \sim ll_p^2$, yielding an average separation between neighboring cells no less than $\sim l^{1/3}l_p^{2/3}$. [9] This spatial separation is interpreted as the average minimum uncertainty in the measurement of a distance l , that is, $\delta l \gtrsim l^{1/3}l_p^{2/3}$, the same result as found above in the Wigner-Salecker gedanken experiment. This result will be shown to be consistent with the holographic principle; hence the corresponding spacetime foam model is called the holographic model.

But there are many other models of spacetime foam [10]. We can characterize them with a parameter $\alpha \sim 1$ according to $\delta l \sim l^{1-\alpha}l_p^\alpha$. It is useful to introduce the following model as a foil to the ($\alpha = 2/3$) holographic model. Instead of maximizing spatial resolution in the mapping of spacetime geometry, let us consider spreading the spacetime cells uniformly in both space and time. In that case, each cell has the size of $(l^2l_p^2)^{1/4} = l^{1/2}l_p^{1/2}$ both spatially and temporally so that each clock ticks once in the time it takes to communicate with a neighboring clock. Since the dependence on $l^{1/2}$ has the hallmark of a random-walk fluctuation, the (quantum foam) model corresponding to $\delta l \gtrsim (ll_p)^{1/2}$ is called the random-walk model [11]. Compared to the holographic model, the random-walk model predicts a coarser spatial resolution, i.e., a larger distance fluctuation, in the mapping of spacetime geometry.² We will concentrate on the holographic model — the only correct model, in my opinion. But occasionally we will consider the general class of models parametrized by the different values of α (specifically only when we discuss the experimental/observational probing of spacetime foam). Unless clarity demands otherwise, we will put $c = 1$ and $\hbar = 1$ henceforth.

Spacetime foam and probing it with distant quasars/AGNs

How can we test the spacetime foam models? The Planck length $l_p \sim 10^{-33}$ cm is so short that we need an astronomical (even cosmological) distance l for its fluctuation δl to be detectable. Thus let us consider light (with wavelength λ) from distant quasars or bright active galactic nuclei [12, 13]. Due to the quantum fluctuations of spacetime, the wavefront, while planar, is itself “foamy”, having random fluctuations in phase [13] $\Delta\phi \sim 2\pi\delta l/\lambda$. When $\Delta\phi \sim \pi$, the cumulative uncertainty in the wave’s phase will have effectively scrambled the wave front sufficiently to prevent the observation

²It also yields a smaller bound on the information content in a spatial region, viz., $(l/l_p)^2/(l/l_p)^{1/2} = (l^2/l_p^2)^{3/4} = (l/l_p)^{3/2}$ bits.

of interferometric fringes. Consider the case of PKS1413+135 [14], an AGN for which the redshift is $z = 0.2467$. With $l \approx 1.2$ Gpc and $\lambda = 1.6\mu\text{m}$, we [13] find $\Delta\phi \sim 10 \times 2\pi$ and $10^{-9} \times 2\pi$ for the random-walk model and the holographic model of spacetime foam respectively. Thus the observation [14] by the Hubble Space Telescope of an Airy ring for this AGN *rules out the random-walk model* but fails to test the holographic model.

Furthermore we [15] note that, due to quantum foam-induced fluctuations in the phase, the wave vector can acquire a cumulative random fluctuation in direction with an angular spread of the order of $\Delta\phi/2\pi$. In effect, spacetime foam creates a “seeing disk” whose angular diameter is $\Delta\phi/(2\pi) \sim (l/\lambda)^{1-\alpha}(l_P/\lambda)^\alpha$ for the model parametrized by α .³

For a telescope or interferometer with baseline length D , this means that dispersion (on the order of $\Delta\phi/2\pi$ in the normal to the wave front) will be recorded as a spread in the angular size of a distant point source, causing a reduction in the Strehl ratio, and/or the fringe visibility when $\Delta\phi/2\pi \sim \lambda/D$, i.e.,

$$(l/\lambda)^{1-\alpha}(l_P/\lambda)^\alpha \sim \lambda/D$$

for a diffraction limited telescope.⁴ Thus, in principle, for arbitrarily large distances spacetime foam sets a lower limit on the observable angular size of a source at a given wavelength λ . Furthermore, the disappearance of “point sources” will be strongly wavelength dependent happening first at short wavelengths. Interferometer systems (like the Very Large Telescope Interferometers when it reaches its design performance) with multiple baselines may have sufficient signal to noise to allow for the detection of quantum foam fluctuations. For a discussion of the constraints recent astrophysical data put on spacetime foam models, see [16].⁵

³This is partly based on the intuition (or reasonable assumption)[15] that spacetime foam fluctuations are isotropic such that the sizes of the wave-vector fluctuations perpendicular to and along the light of sight are comparable. But we should keep in mind that this intuition, though reasonable, could be wrong; after all, spatial isotropy is here “spontaneously” broken with the detected light being from a particular direction.

⁴For example, for a quasar of 1 Gpc away, at an infrared wavelength of the order of 2 microns, the holographic model of spacetime foam predicts a phase fluctuation $\Delta\phi \sim 2\pi \times 10^{-9}$ radians. On the other hand, an infrared interferometer with $D \sim 100$ meters has $\lambda/D \sim 5 \times 10^{-9}$. Such an interferometer has the potential to test the holographic model with a bright enough quasar that distance away.

⁵See Ref. [11, 17] for a discussion of using gravitational-wave interferometers (like LIGO) or laser atom interferometers to detect spacetime foam.

Spacetime foam and turbulence

John Wheeler [18] was among the first to realize the connections between quantum gravity and the ubiquitous phenomenon of turbulence. Due to quantum fluctuations, spacetime, when probed at very small scales, will appear very complicated — something akin in complexity to a chaotic turbulent froth (which, as we all know, he dubbed spacetime foam, also known as quantum foam — the subject matter of this talk.) The connections between quantum gravity and turbulence are quite natural if we recall the role of the (volume preserving) diffeomorphism symmetry in classical (unimodular) gravity and the volume preserving diffeomorphisms of classical fluid dynamics. We may also recall that, in the case of irrotational fluids in three spatial dimensions, the equation for the fluctuations of the velocity potential can be written in a geometric form [19] with a metric having the canonical ADM form [19, 20]. The upshot is that the velocity of the fluid v^i plays the role of the shift vector in Einsteinian gravity; a fluctuation of v^i would imply a quantum fluctuation of the shift vector.

Furthermore, in fully developed turbulence in three spatial dimensions, the remarkable Kolmogorov scaling [21] implies that v scales with length scale l as $\sim l^{1/3}$, consistent with experimental observations. On the other hand, according to the holographic model of spacetime foam, a distance l fluctuates by an amount $\delta l \sim l^{1/3} l_p^{2/3}$. If one defines a velocity as $v \sim \frac{\delta l}{t_c}$, where the natural characteristic time scale is $t_c \sim \frac{l_p}{c}$, then it follows that $v \sim c(l/l_p)^{1/3}$.⁶ Thus we have obtained a *Kolmogorov-like scaling in turbulence, i.e., the velocity scales as*

$$v \sim l^{1/3}.$$

Since the velocities play the role of the shifts, they describe how the metric fluctuates at the Planck scale. The implication is that *at short distances, spacetime is a chaotic and stochastic fluid in a turbulent regime* with the Kolmogorov length l . [22]

Spacetime foam and the holographic principle

In essence, the holographic principle[23, 24, 25] says that although the world around us appears to have three spatial dimensions, its contents can actually be encoded on a two-dimensional surface, like a hologram. In

⁶Here the speed of sound c and the Planck length l_p for an induced gravitational constant are effective quantities.

other words, the maximum entropy, i.e., the maximum number of degrees of freedom, of a region of space is given by its surface area in Planck units. In this section, we will heuristically show that *the holographic principle has its origin in the quantum fluctuations of spacetime*.

Consider partitioning a spatial region measuring l by l by l into many small cubes, with the small cubes being as small as physical laws allow, so that we can associate one degree of freedom with each small cube. [26] In other words, the number of degrees of freedom that the region can hold is given by the number of small cubes that can be put inside that region.

But how small can such cubes be? A moment's thought tells us that each side of a small cube cannot be smaller than the accuracy δl with which we can measure each side l of the big cube. Thus, the number of degrees of freedom (d.o.f.) in the region (measuring l by l by l) is given by $l^3/\delta l^3$, which, since $\delta l \gtrsim l^{1/3}l_p^{2/3}$, is

$$\#d.o.f. \lesssim (l/l_p)^2,$$

as stipulated by the holographic principle. Thus spacetime foam manifests itself holographically.

Spacetime foam and the critical cosmic energy density

Assuming that there is unity of physics connecting the Planck scale to the cosmic scale, we can now apply the holographic spacetime foam model to cosmology [6, 27, 28] and henceforth we call that cosmology the holographic foam cosmology (HFC).

Recall that the minimum δl found for the holographic model corresponds to the case of maximum energy density $\rho = (3/8\pi)(ll_p)^{-2}$ for a sphere of radius l not to collapse into a black hole. Hence the holographic model, unlike the other models, requires, for its consistency, the energy density to have the "critical" value.⁷ Hence, according to HFC, the cosmic energy density is given by

$$\rho = (3/8\pi)(R_H l_p)^{-2},$$

⁷By contrast, for instance, the corresponding energy density for the random-walk model takes on a range of values: $(ll_p)^{-2} \gtrsim \rho \gtrsim l^{-5/2}l_p^{-3/2}$. (The upper bound corresponds to the clocks ticking every $(ll_p)^{1/2}$ while the lower bound corresponds to the clocks ticking only once during the entire time $2l/c$.)

where R_H is the Hubble radius.⁸ This is the critical cosmic energy density as observed.^{9 10} Furthermore, since critical energy density is a hallmark of the inflationary universe scenario, HFC may be consistent with (warm) inflation [33].

Spacetime foam and dark energy/cosmological constant

In this section we will show that HFC “postdicts” the existence of dark energy and yields the correct magnitude of the effective cosmological constant. [6, 27, 28] The argument goes as follows: For the present cosmic era, the energy density is given by $\rho \sim H_0^2/G \sim (R_H l_P)^{-2}$ (about $(10^{-4}eV)^4$), where H_0 is the present Hubble parameter. Treating the whole universe as a computer, one can apply the Margolus-Levitin theorem to conclude that the universe computes at a rate ν up to $\rho R_H^3 \sim R_H l_P^{-2}$ ($\sim 10^{106}$ op/sec), for a total of $(R_H/l_P)^2$ ($\sim 10^{122}$) operations during its lifetime so far. If all the information of this huge computer is stored in ordinary matter, we can apply standard methods of statistical mechanics¹¹ to find that the total number I of bits is $[(R_H/l_P)^2]^{3/4} = (R_H/l_P)^{3/2}$ ($\sim 10^{92}$). It follows that each bit flips once in the amount of time given by $I/\nu \sim (R_H l_P)^{1/2}$ ($\sim 10^{-14}$ sec). However the average separation of neighboring bits is $(R_H^3/I)^{1/3} \sim (R_H l_P)^{1/2}$ ($\sim 10^{-3}$ cm). Hence, assuming only ordinary matter exists to store all the information we are led to conclude that the time to communicate with neighboring bits is equal to the time for each bit to flip once. It follows that the accuracy to which ordinary matter maps out the geometry of spacetime corresponds exactly to the case of events spread out uniformly in space and time as for the random-walk model of spacetime foam.

But, as argued in the introduction, the holographic model, not the random-walk model, is the correct model of spacetime foam. Furthermore, the sharp images of PKS1413+135 observed at the Hubble Space Telescope have ruled out the latter model. From the theoretical as well as observational demise of the random-walk model and the fact that ordinary matter only contains an amount of information dense enough to map out space-

⁸Instead of the Hubble radius, it has been suggested[29, 30] that one should perhaps use the Ricci’s length.

⁹For an alternative explanation of the observed value for ρ , see [31, 32].

¹⁰Note that ρ depends on the geometric mean of R_H , the largest length scale, and l_P , the smallest length scale. This indicates that there is an interplay or connection between ultraviolet and infrared dynamics in HFC and in spacetime foam physics.

¹¹Recall that energy (which determines the number of operations) and entropy (which determines the number of bits) depend on the 4th and 3rd power of temperature respectively.

time at a level consistent with the random-walk model, one now infers that spacetime is mapped to a finer spatial accuracy than that which is possible with the use of ordinary matter. Therefore there must be another kind of substance with which spacetime can be mapped to the observed accuracy, as given by the holographic model. The natural conclusion is that *unconventional (dark) energy/matter exists!* Note that this argument does not make use of the evidence from recent cosmological (supernovae, cosmic microwave background, and galaxy clusters) observations.

Furthermore, the average energy carried by each constituent (particle/bit) of the unconventional energy/matter is $^{12} \sim \rho R_H^3 / I \sim R_H^{-1}$ ($\sim 10^{-31}$ eV). Such long-wavelength (hence “non-local”) constituents of dark energy act as a *dynamical cosmological constant with the observed magnitude* 13

$$\Lambda \sim 3H^2.$$

Thus HFC predicts an accelerating universe. In order to have an earlier decelerating universe and to have a cosmic transition from the decelerating expansion to a recent accelerating expansion, one needs dark matter and probably also an interaction between dark matter and dark energy [34]. 14

Spacetime foam and critical galactic acceleration/MoND

If holographic spacetime foam has provided the cosmos with an effective cosmological constant, one wonders if it may also affect local galactic dynamics. In particular, in view of Verlinde’s recent proposal [35] (see Appendix A) for the entropic [24], and thus holographic [23] reinterpretation of Newton’s law, it is natural to ask: can Newton’s second law be modified by holographic spacetime foam effects?

We first have to recognize that we live in an accelerating universe (in accordance with HFC). This suggests that we will need a generalization [36] of Verlinde’s proposal to de Sitter space with a positive cosmological constant which, according to HFC, is related to the Hubble parameter H by $\Lambda \sim 3H^2$. The Unruh-Hawking temperature [37] as measured by a non-inertial observer with acceleration a in the de Sitter space is given by $\sqrt{a^2 + a_0^2} / (2\pi k_B)$ [38], where $a_0 = \sqrt{\Lambda/3}$ [25]. Consequently, we can define the net temperature measured by the non-inertial observer (relative to the inertial observer) to be $\tilde{T} = [(a^2 + a_0^2)^{1/2} - a_0] / (2\pi k_B)$.

12 Recall that $I \sim (R_H/l_P)^2$ for holographic foam cosmology.

13 For an alternative explanation of the observed magnitude of Λ , see [31, 32].

14 As argued in [34], an appropriate interaction between the two components can even help to alleviate the cosmic coincidence problem.

We can now follow Verlinde's approach[35].¹⁵ Then the entropic force, acting on the test mass m with acceleration a in de Sitter space, is given by $F_{entropic} = \tilde{T} \nabla_x S = m[(a^2 + a_0^2)^{1/2} - a_0]$. For $a \gg a_0$, the entropic force is given by $F_{entropic} \approx ma$. But for $a \ll a_0$, we have $F_{entropic} \approx ma^2/(2a_0)$; and so the terminal velocity v of the test mass m should be determined from $ma^2/(2a_0) = mv^2/r$. [36] The observed flat galactic rotation curves (i.e., at large r , v is independent of r) and the observed Tully-Fisher relation (the speed of stars being correlated with the galaxies' brightness, i.e., $v^4 \propto M$) [39] now require that $a \approx (4a_N a_0^3)^{1/4}$, where $a_N = GM/r^2$ is the magnitude of the usual Newtonian acceleration.¹⁶ But that means $F_{entropic} \approx ma^2/(2a_0) \approx m\sqrt{a_N a_0}$ for the small acceleration $a \ll a_0$ regime. Thus we are led to the modified Newtonian dynamics, or MoND [40], due to Milgrom, which stipulates that the acceleration of a test mass m due to the source M is given by $a = a_N$ and $\sqrt{a_N a_c}$ for $a \gg a_c$ and $a \ll a_c$ respectively ¹⁷ — provided we can identify a_0 as Milgrom's critical acceleration a_c . Milgrom has observed that a_c is numerically related to the speed of light c and the Hubble scale H as ¹⁸ $a_c \sim cH \sim 10^{-8} \text{cm/s}^2$. But $a_0 = (\Lambda/3)^{1/2}$, and $\Lambda \sim 3H^2$ as argued in the last section for HFC, it follows that a_0 is of the order of magnitude of

$$a_{critical} \sim \sqrt{\Lambda/3} \sim H.$$

In other words, *we have successfully predicted the correct magnitude of the critical galactic acceleration, and furthermore have found that global physics (in the form of a dynamical cosmological constant with its origin in spacetime foam) can affect local galactic motion!*

Spacetime foam and cold dark matter with MoND scaling

With only a single parameter (a_c), MoND can explain easily and rather successfully (while the cold dark matter (CDM) paradigm cannot) the ob-

¹⁵We replace the T in Appendix A by \tilde{T} for the Unruh temperature.

¹⁶One can check this by carrying out a simple dimensional analysis and recalling that there are two accelerations in the problem: viz, a_N and a_0 . The factor of $4^{1/4}$ in a is included for convenience only.

¹⁷Our result is not surprising, since MoND has been designed to give the observed flat rotation curves and the Tully-Fisher relation in the first place. Let us also note that actually Milgrom suggested [41] that the generalized Unruh temperature \tilde{T} can give the correct behaviors of the interpolating function between the usual Newtonian acceleration and his suggested MoNDian deformation for very small accelerations. He was right, but he could not offer any justification.

¹⁸To be more precise, $a_c \sim cH/(2\pi)$.

served flat galactic rotation curves^{19 20} and the observed Tully-Fisher relation. But there are problems with MoND at the cluster and cosmological scales, where apparently CDM works much better [43]. This inspires us [36] to ask: Could there be some kind of dark matter that can behave like MoND at the galactic scale?

Let us continue to follow Verlinde’s holographic approach. Invoking the imaginary holographic screen of radius r , we can write²¹ $2\pi k_B \tilde{T} = \frac{G\tilde{M}}{r^2}$, where \tilde{M} represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$. But, as we will show below, consistency with the discussion in the previous section (and with observational data) demands that $\tilde{M} = M + M'$ where M' is some unknown mass — that is, dark matter. Thus, *we need the concept of dark matter for consistency*.

First note that it is natural to write the entropic force $F_{entropic} = m[(a^2 + a_0^2)^{1/2} - a_0]$ as $F_{entropic} = m a_N [1 + 2(a_0/a)^2]$ since the latter expression is arguably the simplest interpolating formula²² for $F_{entropic}$ that satisfies the two requirements: $a \approx (4a_N a_0^3)^{1/4}$ in the small acceleration $a \ll a_0$ regime, and $a = a_N$ in the $a \gg a_0$ regime. But we can also write F in another, yet equivalent, form: $F_{entropic} = mG(M + M')/r^2$. These two forms of F illustrate the idea of CDM-MoND duality.[36] The first form can be interpreted to mean that there is no dark matter, but that the law of gravity is modified, while the second form means that there is dark matter (which, by construction, is consistent with MoND) but that the law of gravity is not modified. The second form gives us this intriguing dark matter profile: $M' = 2 \left(\frac{a_0}{a}\right)^2 M$. Dark matter of this kind can behave as if there is no dark matter but MoND. Therefore, we call it “MoNDian dark matter”. [36] One can solve for M' as a function of r in the two acceleration regimes: $M' \approx 0$ for $a \gg a_0$, and (with $a_0 \sim \sqrt{\Lambda}$)

$$M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$$

for $a \ll a_0$. Intriguingly the dark matter profile we have obtained relates, at the galactic scale,²³ dark matter (M'), dark energy (Λ) and ordinary matter (M)

¹⁹Since the galactic dynamics is very complex, it is not surprising that MoND cannot explain all of the observed galactic velocity curves.

²⁰For other attempts to explain the rotation curves of galaxies, see, e.g., [42]; but typically they all make use of more than one parameter.

²¹We replace the T and M in Appendix A by \tilde{T} and \tilde{M} respectively.

²²But it is not unique – actually, it may be wrong for the $a \sim a_0$ regime.

²³One may wonder why MoND works at the galactic scale, but not at the cluster or cosmic scale. One of reasons is that, for the larger scales, one has to use

to one another.²⁴ As a side remark, this dark matter profile can be used to recover the observed flat rotation curves and the Tully-Fisher relation.

Spacetime foam and nonlocality

According to the holographic principle, the number of degrees of freedom in a region of space is bounded not by the volume but by the surrounding surface. This suggests that the physical degrees of freedom are not independent but, considered at the Planck scale, they must be infinitely correlated, with the result that the spacetime location of an event may lose its invariant significance. *If we take the point of view that holography has its origin in spacetime foam (as we have argued above), then we can argue that spacetime foam gives rise to nonlocality.* This argument is also supported by the following observation [28] that the long-wavelength (hence “non-local”) “particles” constituting dark energy in HFC obey an exotic statistics which has attributes of nonlocality.

Consider a perfect gas of N particles obeying Boltzmann statistics at temperature T in a volume V . For the problem at hand, as the lowest-order approximation, we can neglect the contributions from matter and radiation to the cosmic energy density for the recent and present eras. Then the Friedmann equations for $\rho \sim H^2/G$ can be solved by $H \propto 1/a$ and $a \propto t$, where $a(t)$ is the cosmic scale factor. Thus let us take $V \sim R_H^3$, $T \sim R_H^{-1}$, and $N \sim (R_H/l_P)^2$. A standard calculation (for the relativistic case) yields the partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N$, where $\lambda = (\pi)^{2/3}/T$, and the entropy $S = N[\ln(V/N\lambda^3) + 5/2]$. The important point to note is that, since $V \sim \lambda^3$, the entropy S becomes nonsensically negative unless $N \sim 1$ which is equally nonsensical because $N \sim (R_H/l_P)^2 \gg 1$. The solution comes with the observation that the N inside the log term for S somehow must be absent. Then $S \sim N \sim (R_H/l_P)^2$ without N being small (of order 1) and S is non-negative as physically required. That is the case if the “particles” are distinguishable and nonidentical! For in that case, the Gibbs $1/N!$ factor is absent from the partition function Z_N . Now the only known consistent statistics in greater than two space dimensions

Einstein’s equations with non-negligible contributions from the pressure and explicitly the cosmological constant, which have not been taken into account in the MoND scheme. [36]

²⁴This requires all the three components to exist (an arguably welcome news to HFC) and it indicates possible interactions among them – something, as observed above, that we may need to alleviate the cosmic coincidence problem and to have a cosmic phase transition from a decelerating to an accelerating expansion at redshift $z \sim 1$. [34]

without the Gibbs factor is infinite statistics (sometimes called “quantum Boltzmann statistics”) [44, 45, 46]. (A short description of infinite statistics is given in Appendix B.) Thus we [28] have shown that the “particles” constituting dark energy obey infinite statistics, instead of the familiar Fermi or Bose statistics.²⁵

But it is known that a theory of particles obeying infinite statistics cannot be local [48, 45]. The expression for the number operator

$$n_i = a_i^\dagger a_i + \sum_k a_k^\dagger a_i^\dagger a_i a_k + \sum_l \sum_k a_l^\dagger a_k^\dagger a_i^\dagger a_i a_k a_l + \dots,$$

is both *nonlocal* and nonpolynomial in the field operators, and so is the Hamiltonian. Altogether, the indication is that nonlocality is yet another facet of spacetime foam.^{26 27}

Discussion

In the above sections, we have discussed several facets of spacetime foam. In this section we will mention one *non*-facet of spacetime foam.

Motivated by the interesting detection of a minimal spread in the arrival times of high energy photons from distant GRB reported by Abdo et al.,[50] we can consider using the spread in arrival times of photons as a possible technique for detecting spacetime foam. Now, the spread of arrival times can be traced to fluctuations in the distance that the photons have travelled from the distant source to our telescopes. Hence, according to the spacetime foam model parametrized by α , we get

$$\delta t \sim t^{1-\alpha} t_p^\alpha \sim \delta l / c$$

for the spread in arrival time of the photons, [51] independent of energy E (or photon wavelength λ). Here $t_p \sim 10^{-44}$ sec is the minuscule Planck time. Thus the result is that the time-of-flight differences increase only with the $(1 - \alpha)$ -power of the average overall time of travel $t = l/c$ from the gamma ray bursts to our detector, *leading to a time spread too small to be detectable* (except for the uninteresting range of α close to 0.) The new Fermi Gamma-ray Space Telescope results [50] of $\delta t \lesssim 1$ sec for $t \sim 7$ billion years rule out

²⁵Using the Matrix theory approach, Jejjala, Kavic and Minic [47] have also argued that dark energy quanta obey infinite statistics.

²⁶An interesting question presents itself: Though the nonlocality in holography is probably related to the nonlocality in theories of infinite statistics, how exactly are they related?

²⁷The nonlocal nature of the dynamics of gravitation has been pointed out in other contexts before, see, e.g., [49].

only spacetime foam models with $\alpha \lesssim 0.3$. The holographic model predicts an energy independent dispersion of arrival times $\sim 2.5 \times 10^{-24}$ sec.

Thus we see that, while useful in putting a limit on the variation of the speed of light of a definite sign, this technique is far less useful than the measured angular size in constraining the degree of fuzziness of spacetime in the spacetime foam models. It is easy to understand why that is the case: spacetime foam models predict that the speed of light fluctuates with the fluctuations taking on \pm sign with equal probability; at one instant a particular photon is faster than the average of the other photons, but at the next instant it is slower. The end result is that the cumulative effect due to spacetime foam on the spread in arrival times of photons from distant GRBs is very small (except for spacetime foam models with small α).

Conclusion

Due to the unity of physics, various physical phenomena and structures are inter-related. In this talk I have taken the extreme position of arguing that spacetime foam is the origin of a host of phenomena. For example, the holographic principle finds its roots in spacetime foam physics which also sheds light in explaining why dark energy/dark matter exists. Spacetime foam may explain the observed sizes/magnitudes of the cosmic energy density, the dynamical cosmological constant and the critical galactic acceleration in MoND. It points to the need for cold dark matter with MoND scaling. Possibly spacetime foam is a cause of nonlocal gravitational dynamics. And it has attributes of a turbulent fluid. These are some of the various facets of spacetime foam. Collectively, these facets provide an interesting picture of (and perhaps even some indirect evidence for) it. For completeness, I should add that an *observable* spread in arrival times for (simultaneously emitted) energetic photons from gamma-ray bursts is *not* among the facets of holographic spacetime foam.

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Appendix A: Entropic interpretation of Newton's laws

In this Appendix we review the recent work of E. Verlinde [35] in which the canonical Newton's laws are derived from the point of view of holography. Using the first law of thermodynamics, Verlinde proposes the concept of entropic force $F_{entropic} = T \frac{\Delta S}{\Delta x}$, where Δx denotes an infinitesimal spatial displacement of a particle with mass m from the heat bath with temperature T . He then invokes Bekenstein's original arguments concerning the entropy S of black holes [24] by imposing $\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x$. Using the famous formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer [37], he obtains

$$F_{entropic} = T \nabla_x S = ma,$$

Newton's second law (with the vectorial form $\vec{F} = m\vec{a}$, being dictated by the gradient of the entropy).

Next, Verlinde considers an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T . Then, he assumes the equipartition of energy $E = \frac{1}{2} N k_B T$ with N being the total number of degrees of freedom (bits) on the screen given by $N = A c^3 / (G \hbar)$. Using the Unruh temperature formula and the fact that $E = M c^2$, he obtains

$$2\pi k_B T = GM/r^2$$

and recovers exactly the non-relativistic Newton's law of gravity, namely $a = GM/r^2$. Note that this is precisely the fundamental relation that Milgrom is proposing to modify so as to fit the galactic rotation curves.

Appendix B: Infinite statistics

What is infinite statistics? Succinctly, a Fock realization of infinite statistics²⁸ is given by the average of the commutation relations of the bosonic and fermionic oscillators

$$a_k a_l^\dagger = \delta_{kl}.$$

²⁸More generally, infinite statistics is realized by a q deformation of the commutation relations of the oscillators: $a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{kl}$ with q between -1 and 1 (the case $q = \pm 1$ corresponds to bosons or fermions).[45]

Two states obtained by acting with the N oscillators in different orders are orthogonal. It follows that the states may be in any representation of the permutation group. The statistical mechanics of particles obeying infinite statistics can be obtained in a way similar to Boltzmann statistics, with the crucial difference that the Gibbs $1/N!$ factor is absent for the former. Infinite statistics can be thought of as corresponding to the statistics of identical particles with an infinite number of internal degrees of freedom, which is equivalent to the statistics of nonidentical particles since they are distinguishable by their internal states.

As mentioned in the text, a theory of particles obeying infinite statistics cannot be local [48, 45]. (That is, the fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields.) The expression for the number operator is both nonlocal and nonpolynomial in the field operators, and so is the Hamiltonian. The lack of locality may make it difficult to formulate a relativistic version of the theory; but it appears that a non-relativistic theory can be developed. Lacking locality also means that the familiar spin-statistics relation is no longer valid for particles obeying infinite statistics; hence they can have any spin. Remarkably, the TCP theorem and cluster decomposition have been shown to hold despite the lack of locality. [45]

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